IB PreCalculus Semester Exam Review 2011

Name_____

Roster #_____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Compute the exact value of the function for the given x-value without using a calculator.

1) $f(x) = 6^x$ for $x = 3$

Answer: 216

2) $f(x) = 5^{1-x}$ for $x = 4$

Answer: $\frac{1}{125}$

Describe how the graph of the given function can be obtained by transforming the graph of the reciprocal function $f(x) = 1/x$.

3) $f(x) = \frac{-2}{x + 5}$

Answer: Shift the graph of the reciprocal function left 5 units, reflect across the x-axis, and then stretch vertically by a factor of 2.

Describe how to transform the graph of the basic function g(x) into the graph of the given function f(x).

4) $f(x) = \ln (x + 7) - 6$; $g(x) = \ln x$

Answer: Translate 7 units to the left and 6 units down.

5) $f(x) = - \log (x + 3)$; $g(x) = \log x$

Answer: Translate 3 units to the left then reflect across the y-axis.

Describe the end behavior of the polynomial function by finding $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

6) $f(x) = - (x - 3) (x + 1) (x + 6)^2$

Answer: $-\infty, -\infty$

7) $f(x) = x^3 + x^4 + 6x^2 - 6x + 2$

Answer: ∞, ∞

Determine if the function is bounded above, bounded below, bounded on its domain, or unbounded on its domain.

8) $2^x + 5$

Answer: Bounded below

9) $y = 0$

Answer: Bounded

10) $y = 6 - x^2$

Answer: Bounded above

Determine the doubling time of the investment.

11) 5.25% APR compounded annually

Answer: 13.55 years

12) 8.25% APR compounded semiannually

Answer: 8.57 years

Determine the order of the matrix.

13) $\begin{bmatrix} 3 & -5 & 1 & 7 \end{bmatrix}$

Answer: 1 × 4

14) [4]

Answer: 1 × 1

Determine whether the equation defines y as a function of x.

15) $y = x^2 + 8$

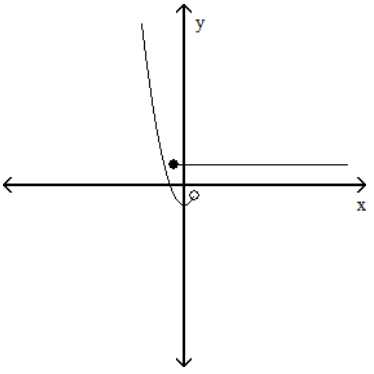
Answer: Yes

16) $y = -2x - 9$

Answer: Yes

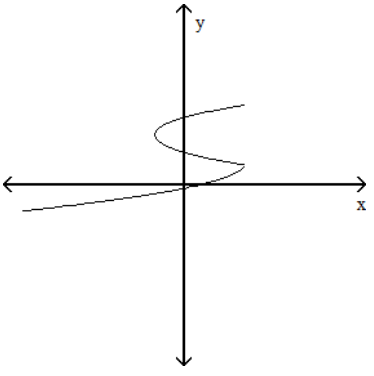
Determine whether the graph is the graph of a function.

17)



Answer: No

18)



Answer: No

Determine whether the infinite geometric series converges. If the series converges, determine the limit.

19) 7 + 21 + 63 + 189 + ...

Answer: Diverges

Determine whether the sequence converges or diverges. If it converges, give the limit.

20) $72, -12, 2, -\frac{1}{3}, \ldots$

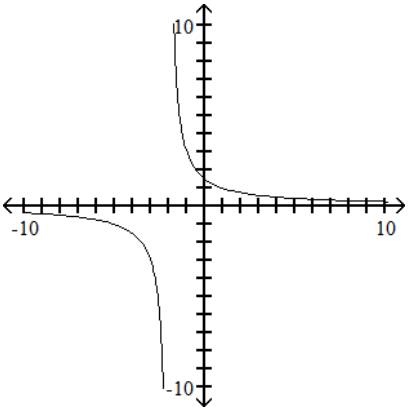
Answer: Converges; $\frac{432}{7}$

21) $72, 18, \frac{9}{2}, \frac{9}{8}, \ldots$

Answer: Converges; 0

Evaluate the limit based on the graph of f shown.

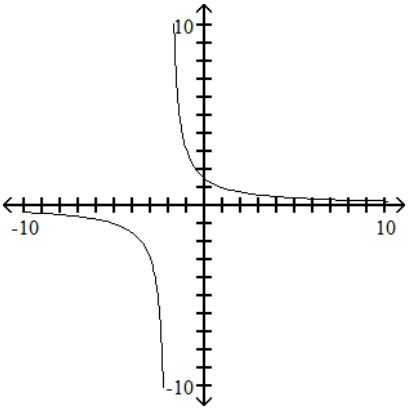
22)



$\lim_{x \to -2^+} f(x)$

Answer: ∞

23)



$\lim_{x \to \infty} f(x)$

Answer: 0

Evaluate the logarithm.

24) $\log_3(\frac{1}{3})$

Answer: -1

25) $\log_{18}\sqrt{18}$

Answer: $\frac{1}{2}$

Fill in the blanks to complete the statement.

26) The graph of $y = -\sqrt{x+8}$ can be obtained from the graph of $y = \sqrt{x}$ by shifting horizontally ? units to the ? and reflecting across the ? -axis.

Answer: 8; left; x

27) The graph of $y = 0.7|-x| - 4$ can be obtained from the graph of $y = |x|$ by reflecting across the ? -axis, shrinking vertically by a factor of ?, and then shifting vertically ? units in the ? direction.

Answer: y; 0.7; 4; downward

Find a matrix A and a column matrix B that describe the following tables involving credits and tuition costs. Find the matrix product AB, and interpret the significance of the entries of this product.

28)

Credits	College A	College B
Student 1	4	11
Student 2	6	6

Cost	Tuition
College A	$86
College B	$65

Answer:

$$AB = \begin{bmatrix} 1059 \\ 906 \end{bmatrix}$$

Tuition for Student 1 is $1059 and tuition for Student 2 is $906.

Find a recursive rule for the nth term of the sequence.

29) -6, 3, 12, 21, ...

Answer: $a_n = a_{n-1} + 9$

Find an explicit rule for the nth term of the arithmetic sequence.

30) -21, -26, -31, -36, ...

Answer: $a_n = -21 + (-5)(n-1)$

31) -1, -3, -5, -7, ...

Answer: $a_n = -1 + -2(n-1)$

Find an explicit rule for the nth term of the sequence.

32) The second and fifth terms of a geometric sequence are -2 and 16, respectively.

Answer: $a_n = 1 \cdot (-2)^{n-1}$

33) The second and fifth terms of a geometric sequence are 6 and 48, respectively.

Answer: $a_n = 3 \cdot 2^{n-1}$

Find functions f and g so that h(x) = f(g(x)).

34) $y = \dfrac{1}{x^2 - 3}$

Answer: f(x) = 1/x, $g(x) = x^2 - 3$

Find the amount accumulated after investing a principal P for t years at an interest rate r.

35) P = $2365, t = 9 , r = 5.3% , compounded quarterly (k = 4)

Answer: $3798.65

36) P = $5073.34, t = 10 , r = 8% , compounded monthly (k = 12)

Answer: $11,260.99

Find the asymptote(s) of the given function.

37) $g(x) = \dfrac{x+7}{x^2 - 1}$ horizontal asymptotes(s)

Answer: y = 0

38) $f(x) = \dfrac{4x^2 + 7}{4x^2 - 7}$ horizontal asymptotes(s)

Answer: y = 1

39) $f(x) = 4.9^x$ horizontal asymptotes(s)

Answer: y = 0

Find the axis of the graph of the function.

40) $f(x) = 4x^2 + 16x + 21$

Answer: x = -2

41) $f(x) = 2x^2 + 20x + 54$

Answer: x = -5

Find the domain of the function.

42) $f(x) = \ln(2x - x^2)$

Answer: (0, 2)

43) $f(x) = \log_7(3x - 3)$

Answer: $(1, \infty)$

Find the domain of the given function.

44) $f(x) = \dfrac{\sqrt{9 - x^2}}{x - 1}$

Answer: $[-3, 1) \cup (1, 3]$

45) $f(x) = \sqrt{x^4 - 64x^2}$

Answer: $(-\infty, -8] \cup [0] \cup [8, \infty)$

Find the exact solution to the equation.

46) $\log_9 x = 5$

Answer: $x = 9^5$

47) $2^{(12 - 2x)} = 256$

Answer: $x = 2$

Find the first six terms of the sequence.

48) $a_1 = -1, a_n = 4 \cdot a_{n-1}$

Answer: $-1, -4, -16, -64, -256, -1024$

49) $a_1 = 4, a_2 = 5$; for $n \geq 3$, $a_n = a_{n-1} + a_{n-2}$

Answer: 4, 5, 9, 14, 23, 37

Find the indicated matrix product or state that the product is undefined.

50) $A = \begin{bmatrix} -1 \\ -2 \end{bmatrix}, B = \begin{bmatrix} -6 & -8 \\ -5 & 3 \end{bmatrix}$

AB

Answer: Undefined

51) $A = \begin{bmatrix} -2 & 7 \\ -6 & -3 \end{bmatrix}, B = \begin{bmatrix} -7 & 7 & 6 \\ -2 & 9 & -9 \end{bmatrix}$

BA

Answer: Undefined

Find the indicated matrix.

52) Let $A = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 6 \end{bmatrix}$. Find $3A + B$.

Answer:

$$\begin{bmatrix} 6 & 13 \\ 5 & 21 \end{bmatrix}$$

53) Let $C = \begin{bmatrix} 2 \\ -2 \\ 10 \end{bmatrix}$. Find $\frac{1}{2}C$.

Answer:

$$\begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}$$

Find the inverse of the function.

54) $f(x) = \sqrt{x - 6}$

Answer: $f^{-1}(x) = x^2 + 6, x \geq 0$

55) $f(x) = 5x + 6$

Answer: $f^{-1}(x) = \dfrac{x - 6}{5}$

Find the logistic function that satisfies the given conditions.

56) Initial value = 20, limit to growth = 80, passing through (1, 32)

Answer: $f(x) = \dfrac{80}{1 + 3(1/2)^x}$

Find the matrix product, if possible.

57) $\begin{bmatrix} 3 & -1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 2 & 6 \end{bmatrix}$

Answer:

$$\begin{bmatrix} -2 & -9 \\ 0 & -3 \end{bmatrix}$$

58) $\begin{bmatrix} -1 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

Answer:

$$\begin{bmatrix} -1 & 3 \\ -6 & 2 \end{bmatrix}$$

59) $\begin{bmatrix} -5 & 4 & -2 \end{bmatrix}\begin{bmatrix} -2 & 2 & -4 \\ 2 & -1 & 5 \\ -8 & -7 & -2 \end{bmatrix}$

Answer:

[34 0 44]

Find the range of the function.

60) $f(x) = 7 - x^2$

Answer: $(-\infty, 7]$

61) $f(x) = 7x - 11$

Answer: $(-\infty, \infty)$

Find the sum of the arithmetic sequence.

62) 221, 219, 217, 215, ..., 203

Answer: 2120

63) 40, 42, 44, 46, ..., 62

Answer: 612

Find the sum of the first n terms of the sequence.

64) $-5, -20, -80, \ldots ; n = 9$

Answer: - 436905

65) $4, -8, 16, \ldots ; n = 11$

Answer: 2732

Find the sum of the geometric sequence.

66) 2, -4, 8, -16, 32

Answer: 22

67) 1, 3, 9, 27, 81

Answer: 121

Find the vertex of the graph of the function.

68) $f(x) = (x + 5)^2 + 1$

Answer: (-5, 1)

69) $f(x) = 8x^2 - 160x + 803$

Answer: (10, 3)

Find the zeros of the function.

70) $f(x) = x^2 + 9x + 20$

Answer: -4 and -5

71) $f(x) = x^3 + 9x^2 + 26x + 24$

Answer: -3, -2, and -4

For the given function, find all asymptotes of the type indicated (if there are any)

72) $f(x) = \dfrac{(x - 8)(x + 4)}{x^2 - 1}$, vertical

Answer: $x = 1, x = -1$

73) $f(x) = \dfrac{8x^2 - 2x - 2}{3x^2 - 7x + 6}$, horizontal

Answer: $y = 8/3$

Give the equation of the function g whose graph is described.

74) The graph of $f(x) = \sqrt{x}$ is shifted 4 units to the left. Then the graph is shifted 2 units upward.

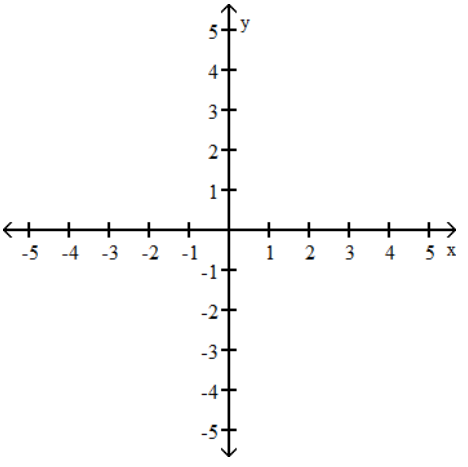
Answer: $g(x) = \sqrt{x + 4} + 2$

75) The graph of $f(x) = x^2 - 4x + 3$ is horizontally shrunk by a factor of 1/4 .

Answer: $g(x) = 4x^2 - 16x + 12$

Graph the function and determine if it has a point of discontinuity at x = 0. If there is a discontinuity, tell whether it is removable or non-removable.

76) $f(x) = \dfrac{|x + 4|}{x}$



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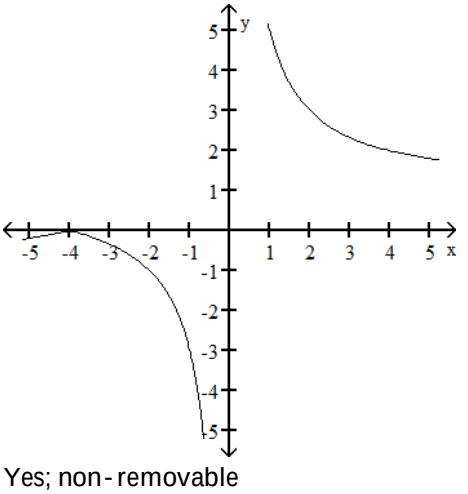
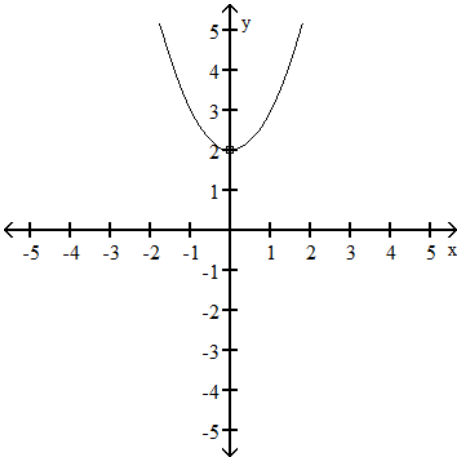


Yes; non-removable

77) $f(x) = \dfrac{x^3 + 2x}{x}$

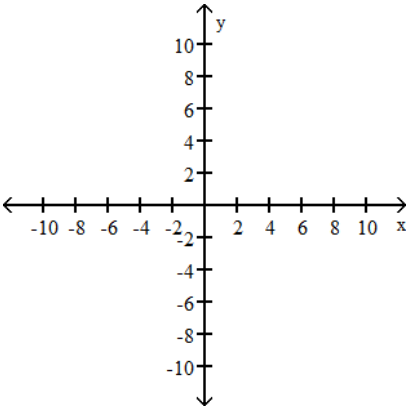
Answer:



Yes; removable

Graph the function.

78) $f(x) = \dfrac{9}{1 + 4e^{-x}}$



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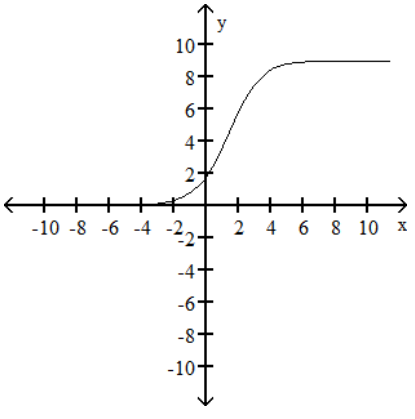
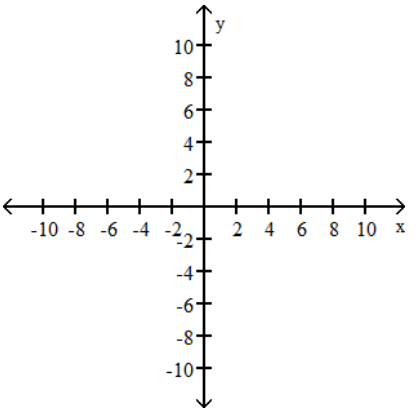


79) $f(x) = \dfrac{4}{1 + 2 \cdot 0.6^x}$



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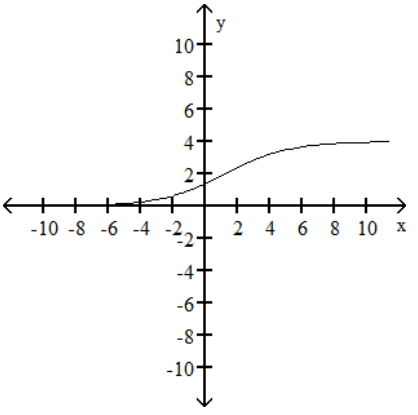
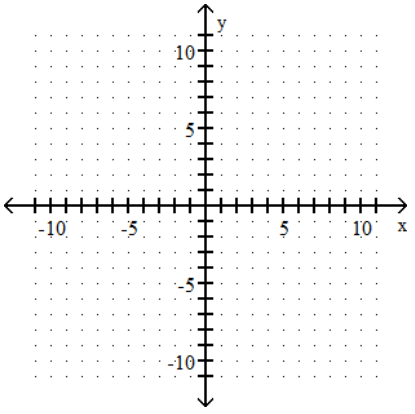
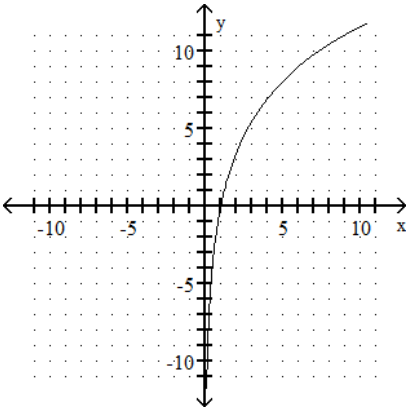


Graph the function. Describe its position relative to the graph of the indicated basic function.

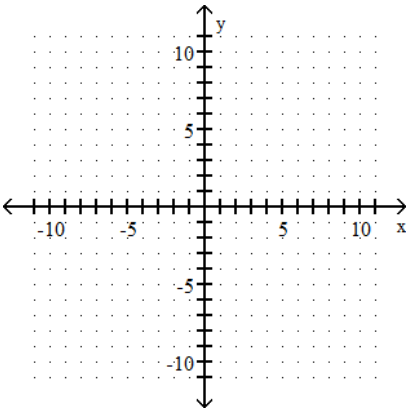
80) f(x) = 5ln x; relative to f(x) = ln x



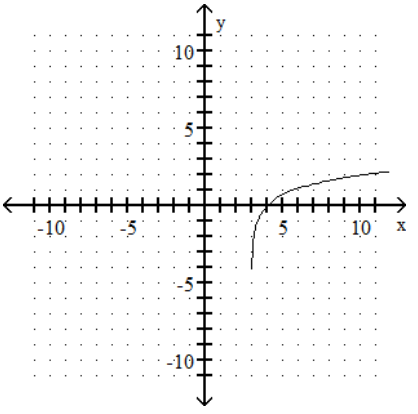
Answer: Stretched vertically



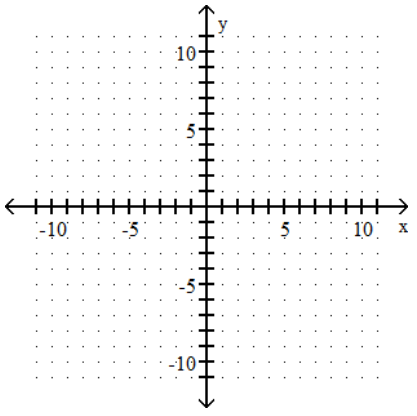
81) f(x) = ln(x - 3); relative to f(x) = ln x



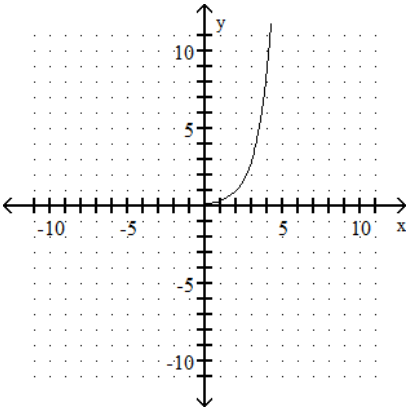
Answer: Moved right 3 units



82) $f(x) = 3^{x-2}$; relative to $f(x) = 3^x$

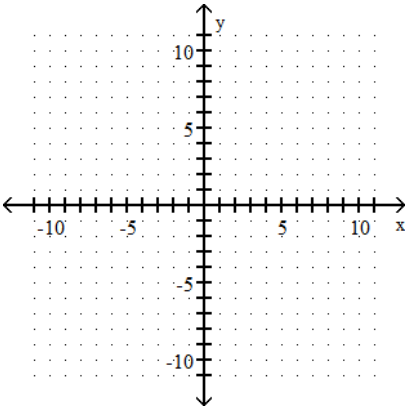


Answer: Moved right 2 unit(s)



Graph the piecewise-defined function.

83) $f(x) = \begin{cases} \sin x \text{ if } x \le 0 \\ |x| \text{ if } x > 0 \end{cases}$



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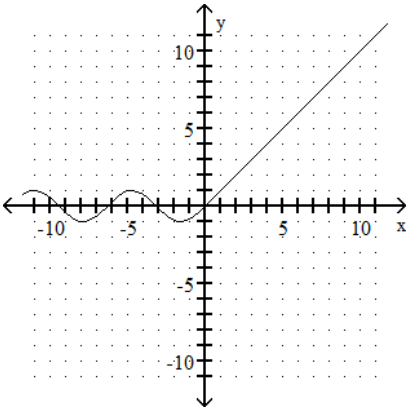
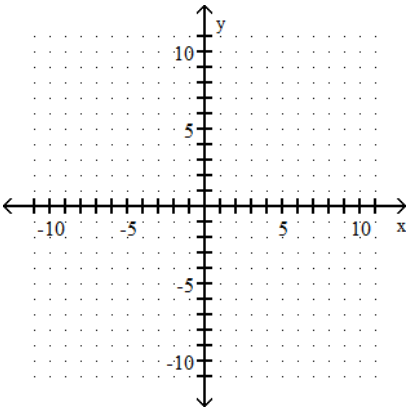


84)

$$f(x) = \begin{cases} |x| + 4, \text{ if } x < 0 \\ 4, \text{ if } x \geq 0 \end{cases}$$



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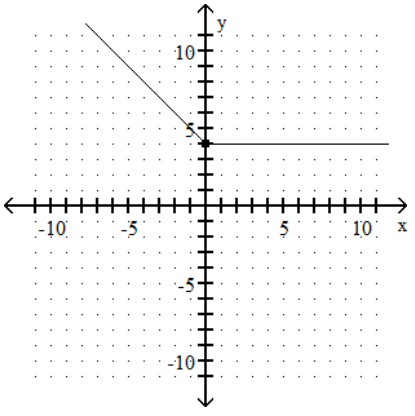


Identify intervals on which the function is increasing, decreasing, or constant.

85) $f(x) = |x - 7| - 5$

Answer: Increasing: $(7, \infty)$; decreasing: $(-\infty, 7)$

Identify which of the twelve basic functions listed below fit the description given.

$y = x$, $y = x^2$, $y = x^3$, $y = |x|$, $y = \frac{1}{x}$, $y = e^x$, $y = \sqrt{x}$, $y = \ln x$, $y = \sin x$, $y = \cos x$, $y = \text{int}(x)$, $y = \frac{1}{1 + e^{-x}}$

86) The three functions that are bounded above

Answer: $y = \sin x$, $y = \cos x$, $y = \frac{1}{1 + e^{-x}}$

87) The two functions with infinitely many zeros

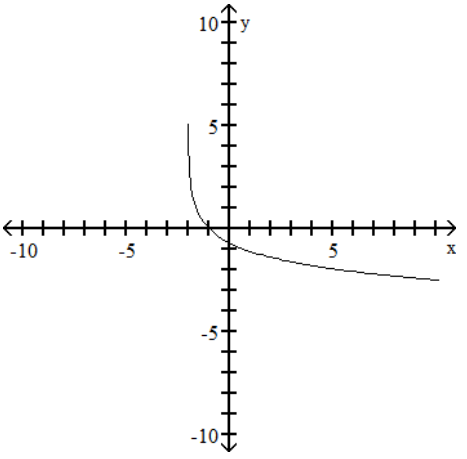
Answer: $y = \sin x$, $y = \cos x$

88) The four functions with local minima

Answer: $y = x^2$, $y = \sin x$, $y = \cos x$, $y = |x|$

Match the function with the graph.

89)



Answer: $y = -\ln(x + 2)$

Perform the indicated operation, if possible.

90) $\begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix} + \begin{bmatrix} -5 \\ 4 \\ 6 \end{bmatrix}$

Answer:

$$\begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$$

91) $\begin{bmatrix} 3 & 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

Answer: Not defined

Perform the requested operation or operations.

92) $f(x) = \frac{1}{x - 4}$; $g(x) = \sqrt{x}$

Find $f(g(x))$.

Answer: $f(g(x)) = \frac{1}{\sqrt{x} - 4}$

93) $f(x) = x^2 + 2$; $g(x) = \sqrt{x - 5}$

Find $f(g(x))$.

Answer: $f(g(x)) = x - 3$

Rewrite the expression as a sum or difference or multiple of logarithms.

94) $\log_{14} x^{14}$

Answer: $14 \log_{14} x$

95) $\log_8 12x$

Answer: $\log_8 12 + \log_8 x$

Simplify the expression.

96) $15^{\log_{15} 8}$

Answer: 8

97) $10^{\log(1/7)}$

Answer: 1/7

Solve the equation algebraically.

98) $v^2 + 5 = 7 - 2v^2$

Answer: $\pm\sqrt{\frac{2}{3}}$

99) $x(x - 0) = 4$

Answer: $-2; 2$

Solve the equation by changing it to exponential form.

100) $\log x = -3$

Answer: $x = \frac{1}{10^3}$

101) $\log_8 x = 2$

Answer: $x = 8^2$

Solve the equation.

102) $\log(4 + x) - \log(x - 2) = \log 3$

Answer: 5

103) $e^x + e^{-x} = 5$

Answer: 1.5668, -1.5668

104) $\log 5x = \log 3 + \log(x + 2)$

Answer: 3

105) $512^x = 8$

Answer: 1/3

106) $(5/6)^x = \frac{216}{125}$

Answer: -3

Solve the problem.

107) An earthquake was recorded which was 501,187 times more powerful than a reference level zero earthquake. What is the magnitude of this earthquake on the Richter scale? Intensity on the Richter scale is $\log_{10}(I/I_0)$.

Answer: 5.7

108) Use the formula $M = \log(A/A_0)$, where the magnitude of an earthquake on the Richter scale is based on A, the measurement of a seismic wave, and A_0, the measurement of a seismic wave of a level zero earthquake with the same epicenter. If an earthquake measured 7.9 on the Richter scale, what was the intensity of the earthquake? (Round to two significant digits.)

Answer: 79,000,000

109) Wind speed varies in the first twenty meters above the ground. For a particular day, let $f(x) = 4.5\ln x + 6.5$ compute the wind speed x meters above the ground. At what height is the wind speed 9 meters per second? Round results to the nearest hundredth.

Answer: 1.75

110) Find a natural logarithmic regression equation for the following data and use it estimate the production level for the year 1985. (For the regression equation, assume t = 0 is the year 1950.)

Year	Production (in millions)
1960	9.26
1970	51.43
1990	68.91

Answer: 67.28

111) The number of students infected with the flu on a college campus after t days is modeled by the function $P(t) = \frac{200}{1 + 39e^{-0.3t}}$. What was the initial number of infected students?

Answer: 5

112) Use the data in the table to compute a logistic regression model for the population of the city t years after 1900.

Year	Pop (in millions)
1900	0.8
1910	1.0
1920	1.3
1930	1.7
1940	2.0
1950	2.5
1960	3.0
1970	3.6

Answer: $P(t) = \dfrac{9.053}{1 + 10.279e^{-0.0273t}}$

113) The number of books in a small library increases according to the function $B = 7800e^{0.03t}$, where t is measured in years. How many books will the library have after 7 years?

Answer: 9623

114) There are currently 68 million cars in a certain country, increasing by 1.8% annually. How many years will it take for this country to have 81 million cars? Round to the nearest year.

Answer: 10 yr

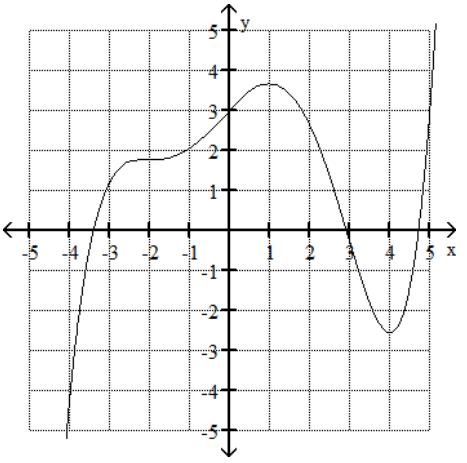
115) In September 1998 the population of the country of West Goma in millions was modeled by $f(x) = 17.5e^{0.0018x}$. At the same time the population of East Goma in millions was modeled by $g(x) = 13.4e^{0.0103x}$. In both formulas x is the year, where x = 0 corresponds to September 1998. Assuming these trends continue, estimate what the population will be when the populations are equal.

Answer: 19 million

116) The decay of 220 mg of an isotope is given by $A(t) = 220e^{-0.016t}$, where t is time in years. Find the amount left after 82 years.

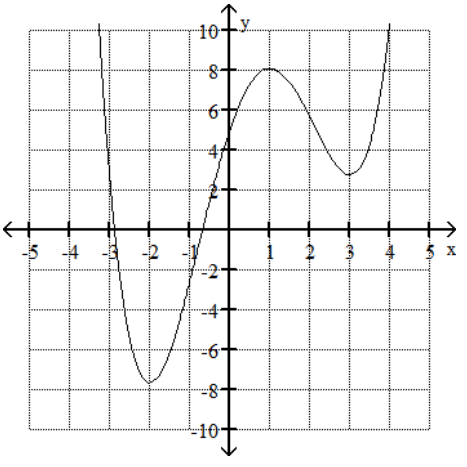
Answer: 59

117) Use the graph of f to estimate the local maximum and local minimum.



Answer: Local maximum: approx. 3.66; local minimum: approx. -2.55

118) Use the graph of f to estimate the local maximum and local minimum.



Answer: Local maximum: approx. 8.08; local minima: approx. -7.67 and 2.75

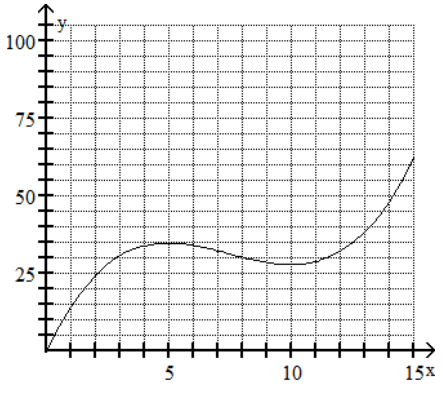
119) The results of an analysis, on the makeup of garbage, done by the Environmental Protection Agency was published in 1990. Some of the results are given in the following table, which for various years gives the number of pounds per person per day of various types of waste materials.

Waste materials	1960	1970	1980	1988
Glass	.20	.34	.36	.28
Plastics	.01	.08	.19	.32
Metals	.32	.38	.35	.34
Paper	.91	1.19	1.32	1.60

For glass, calculate the average rate of change between 1980 and 1988. Then interpret what this value means.

Answer: From 1980 to 1988, the number of pounds of glass per person per day decreased by .01 per year.

120) The graph depicts a person's speed y, in miles per hour, during a 15-minute period of driving.
The graph has two turning points.The first turning point is the point at which the graph stops rising and starts to fall. The second turning point is the point at which the graph stops falling and starts to rise again.
Estimate and interpret the turning points.



Answer: The first turning point is at approximately (5, 35). This is where the person's speed first stops increasing and starts to decrease. The second turning point is at approximately (10, 28). This is where the person's speed stops decreasing and starts to increase again.

Solve.

121) A certain radioactive isotope has a half-life of 7 days. If one is to make a table showing the half-life decay of a sample of this isotope from 32 grams to 1 gram; list the time (in days, starting with t=0) in the first column and the mass remaining (in grams) in the second column, which type of sequence is used in the first column and which type of sequence is used in the second column?

Answer: Arithmetic in the first; geometric in the second

State the domain of the rational function.

122) $f(x) = \dfrac{x - 1}{x^2 + 3}$

Answer: $(-\infty, \infty)$

123) $f(x) = \dfrac{x - 7}{x^2 - 25}$

Answer: $(-\infty, -5) \cup (-5, 5) \cup (5, \infty)$

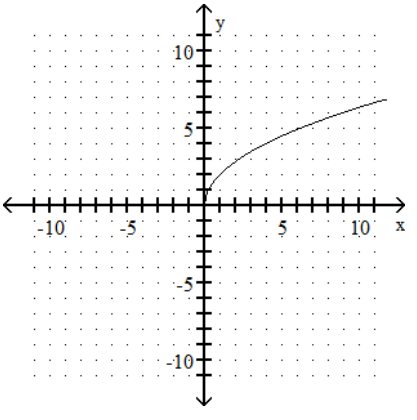
State whether the function is an exponential growth function or exponential decay function, and describe its end behavior using limits.

124) $f(x) = 0.2^x$

Answer: Exponential decay function; $\lim_{x \to -\infty} f(x) = \infty$; $\lim_{x \to \infty} f(x) = 0$

The graph is that of a function y = f(x) that can be obtained by transforming the graph of $y = \sqrt{x}$. Write a formula for the function f.

125)



Answer: $f(x) = 2\sqrt{x}$

Use limits to describe the behavior of the rational function near the indicated asymptote.

126) $f(x) = \dfrac{1}{x^2 - 9}$

Describe the behavior of the function near its vertical asymptotes.

Answer: $\lim\limits_{x \to -3^-} f(x) = \infty$, $\lim\limits_{x \to -3^+} f(x) = -\infty$,
$\lim\limits_{x \to 3^-} f(x) = -\infty$, $\lim\limits_{x \to 3^+} f(x) = \infty$

127) $f(x) = \dfrac{4x^2 - 3}{x^2 + 1}$

Describe the behavior of the function near its horizontal asymptote (the end behavior).

Answer: $\lim\limits_{x \to -\infty} f(x) = 4$, $\lim\limits_{x \to \infty} f(x) = 4$

Use the change of base rule to find the logarithm to four decimal places.

128) $\log_7 20.19$

Answer: 1.5444

129) $\log_6 0.525$

Answer: -0.3596

Use the product, quotient, and power rules of logarithms to rewrite the expression as a single logarithm. Assume that all variables represent positive real numbers.

130) $5 \log_m q - 7 \log_m x^2$

Answer: $\log_m \dfrac{q^5}{x^{14}}$

131) $\dfrac{1}{6} \ln x$

Answer: $\ln \left(\sqrt[6]{x} \right)$

Write an equation for the quadratic function whose graph contains the given vertex and point.

132) Vertex (5, -2), point (-4, 241)

Answer: $P(x) = 3x^2 - 30x + 73$

133) Vertex (-5, 2), point (1, 110)

Answer: $P(x) = 3x^2 + 30x + 77$

Write out the first five terms of the sequence.

134) $a_n = n - 4$

Answer: -3, -2, -1, 0, 1

135) $c_n = \dfrac{n+2}{3}$

Answer: $1, \dfrac{4}{3}, \dfrac{5}{3}, 2, \dfrac{7}{3}$

Write the quadratic function in vertex form.

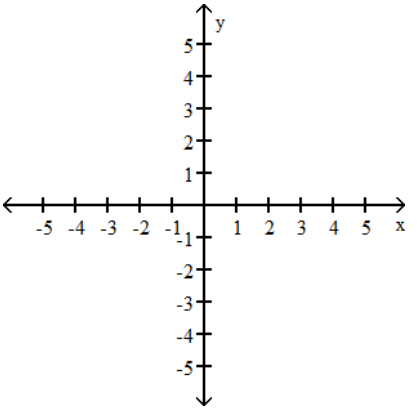
136) $y = x^2 + 6x + 1$

Answer: $y = (x + 3)^2 - 8$

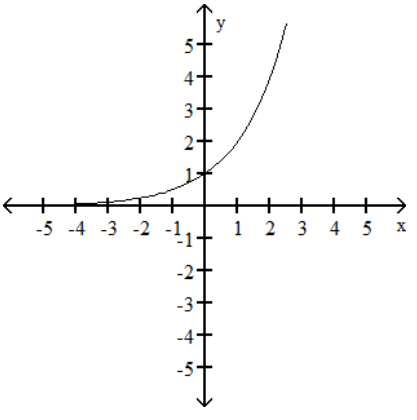
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Choose the graph which matches the function.

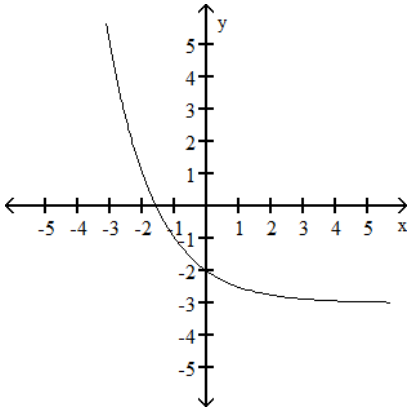
137) $f(x) = 2^{-x} - 3$



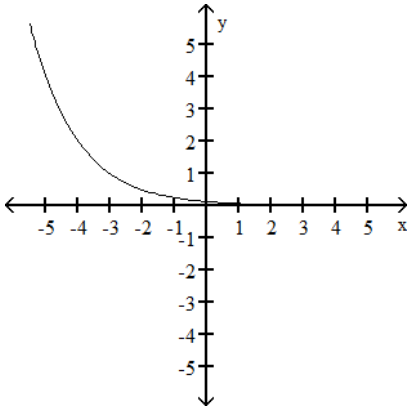
A)



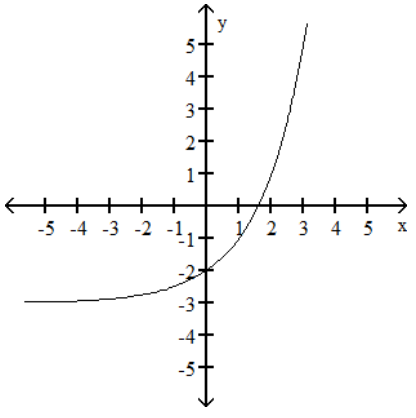
B)



C)

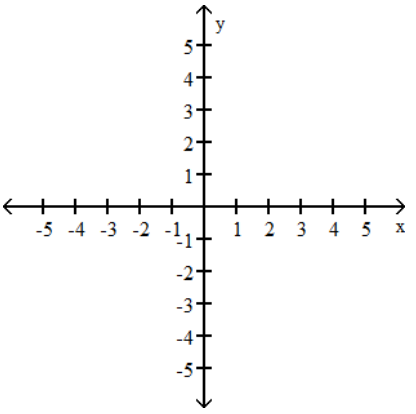


D)

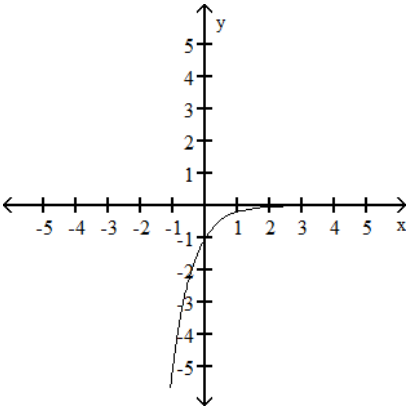


Answer: B

138) $f(x) = -5^x$



A)



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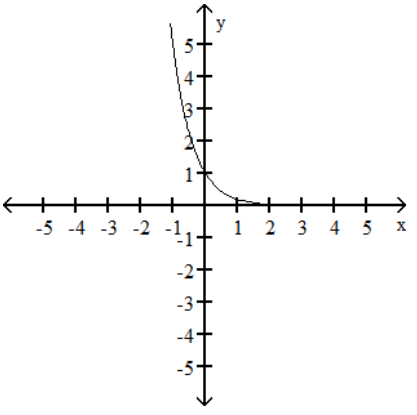
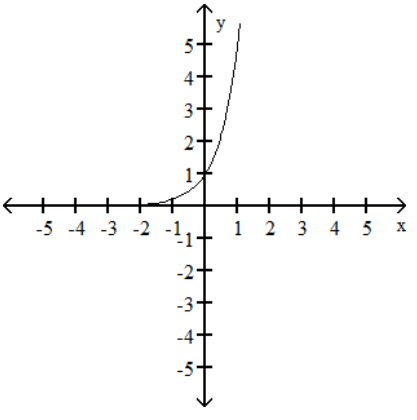
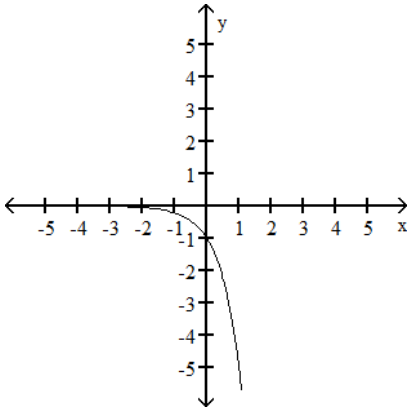


C)



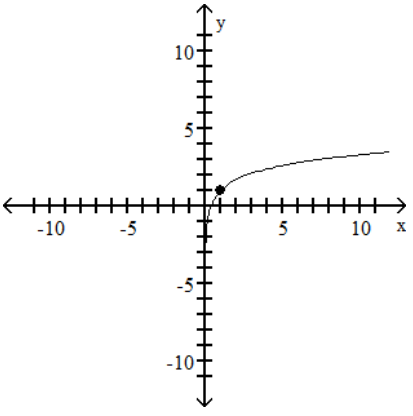
D)



Answer: D

Determine the function which corresponds to the given graph.

139)



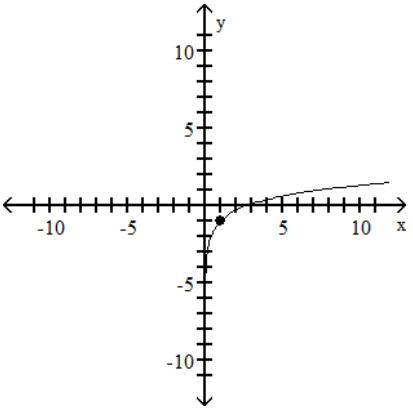
The indicated point is (1, 1).

A) y = ln x - 1 B) y = ln(x + 1)

C) y = ln x + 1 D) y = ln(x - 1)

Answer: C

140)



The indicated point is (1, -1).

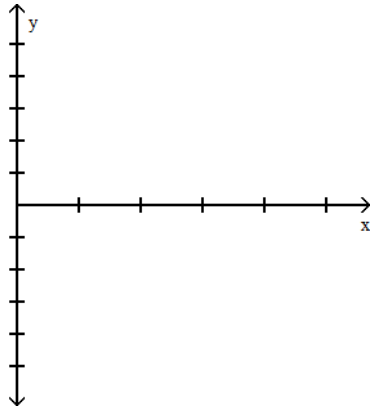
A) y = ln(x - 1) B) y = ln x - 1

C) y = ln x + 1 D) y = ln(x + 1)

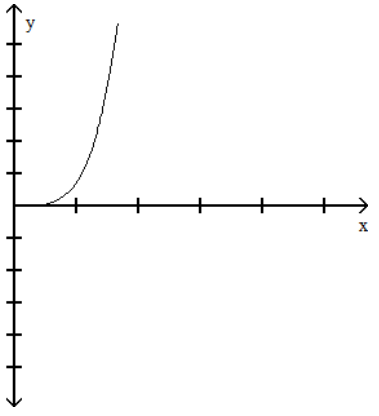
Answer: B

Match the equation to one of the curves (for $x \geq 0$).

141) $f(x) = \frac{3}{4}x - 5$



A)



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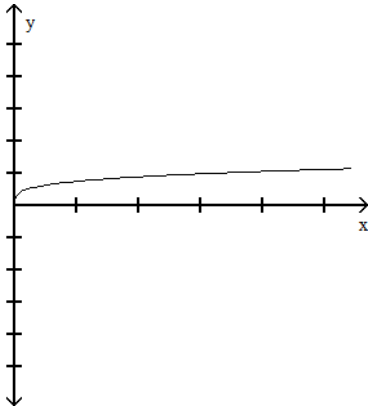
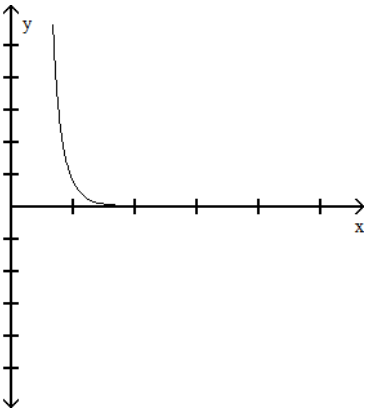
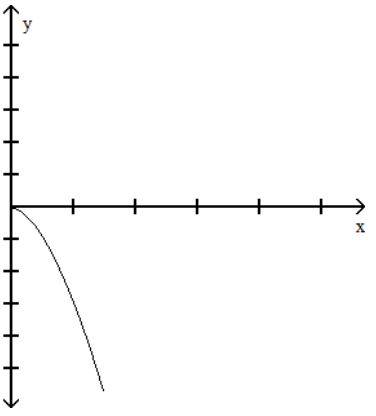


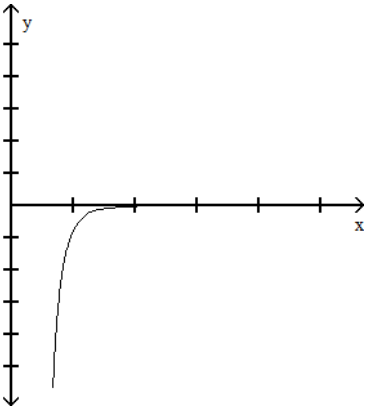
C)



A)



D)



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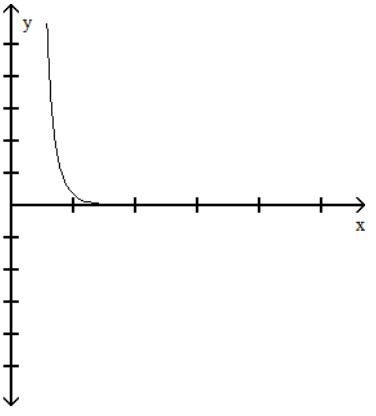
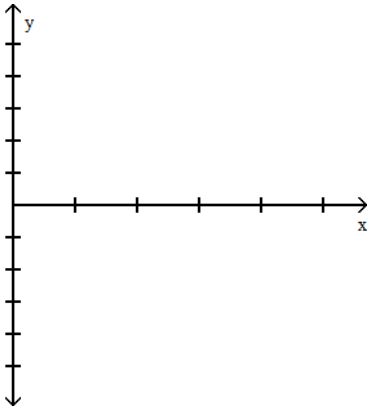
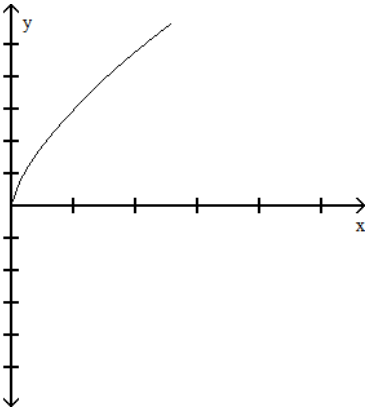


Answer: C

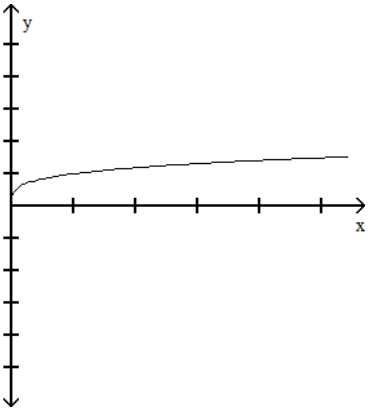
142) $f(x) = -3x^{5/3}$



C)



D)

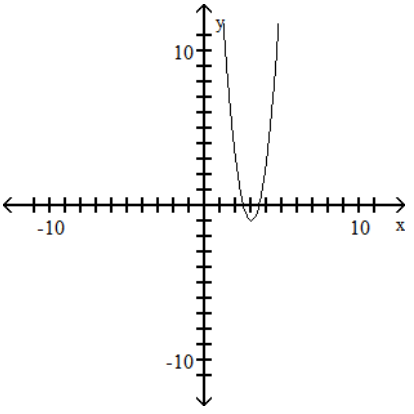


Answer: A

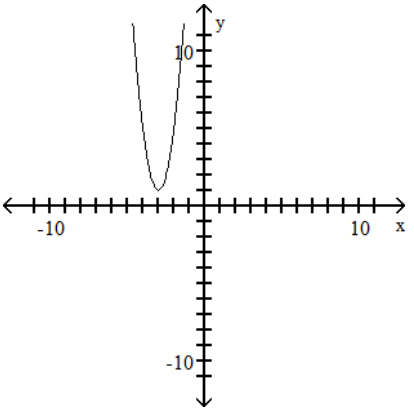
Match the equation to the correct graph.

143) $f(x) = 4(x + 3)^2 - 1$

A)



B)



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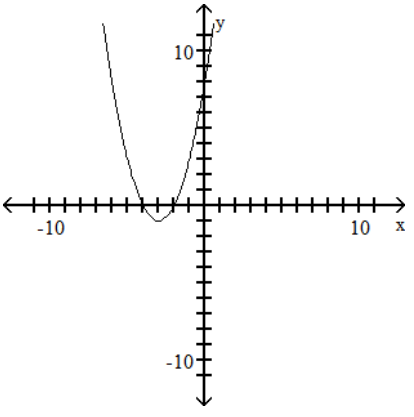
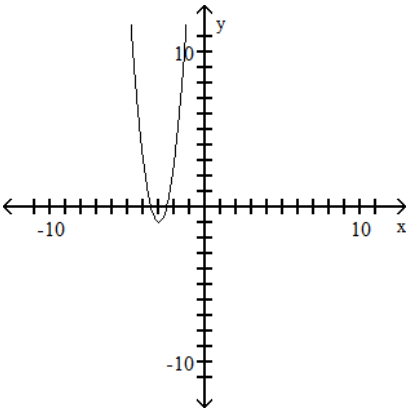


D)

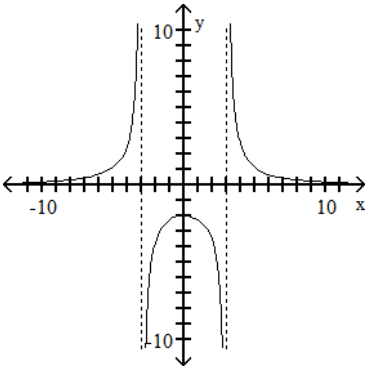


Answer: D

Match the equation with the appropriate graph.

144) $f(x) = \dfrac{18x}{x^2 - 9}$

A)



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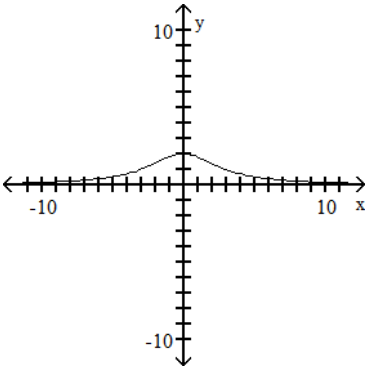

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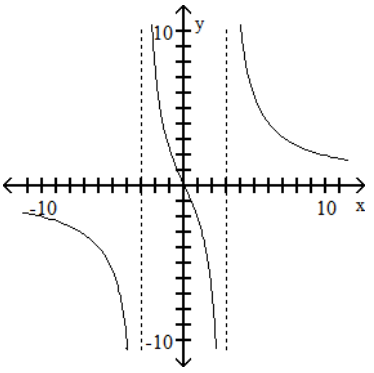
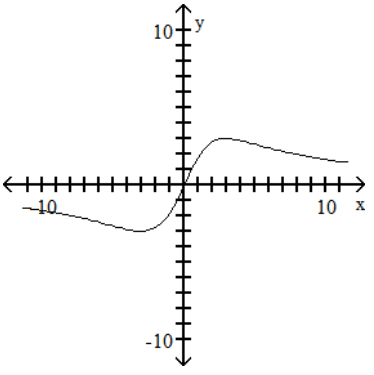
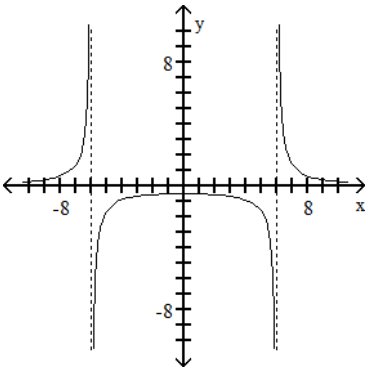


D)

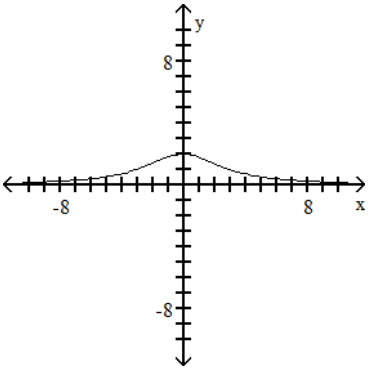


Answer: C

C)



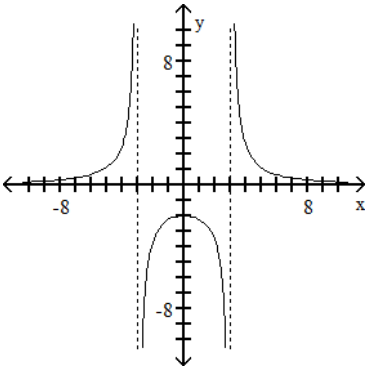
D)



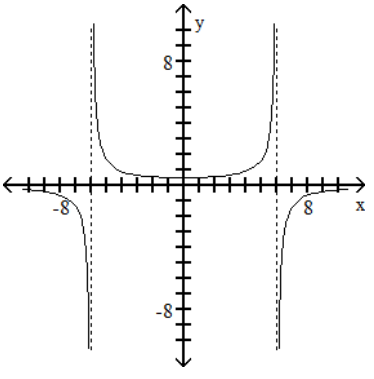
Answer: D

145) $f(x) = \frac{18}{x^2 + 9}$

A)



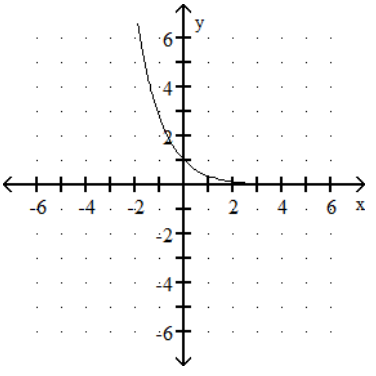
B)



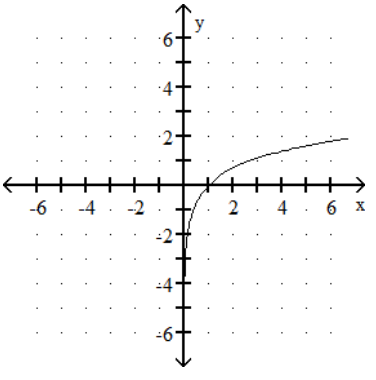
Match the function f with its graph.

146) $f(x) = \ln(x)$

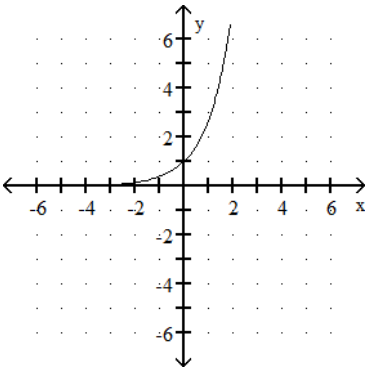
A)



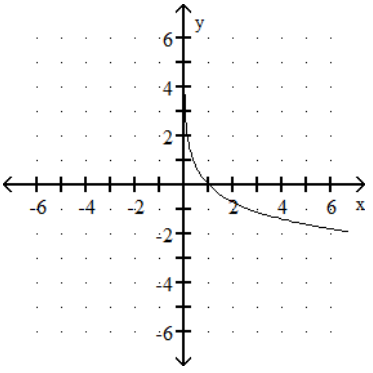
B)



C)

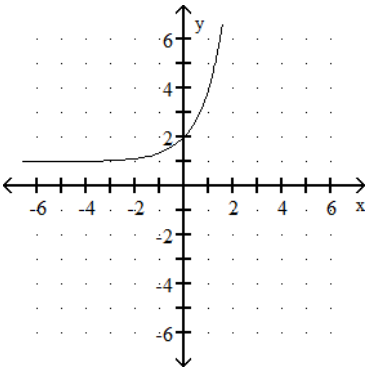


D)

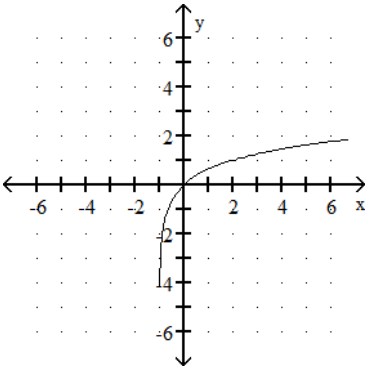


Answer: B

C)



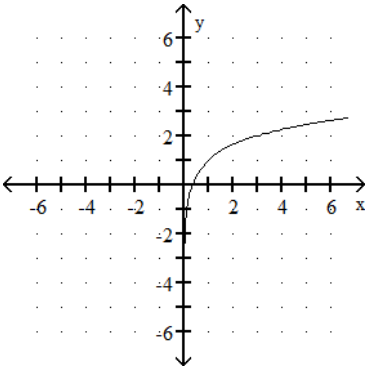
D)



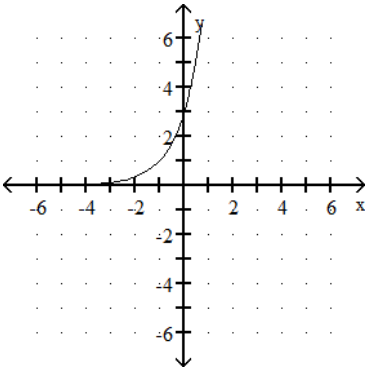
Answer: A

147) $f(x) = \log_3 (x) + 1$

A)

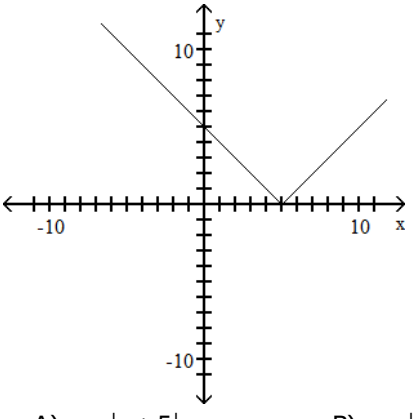


B)



Match the function with the graph.

148)

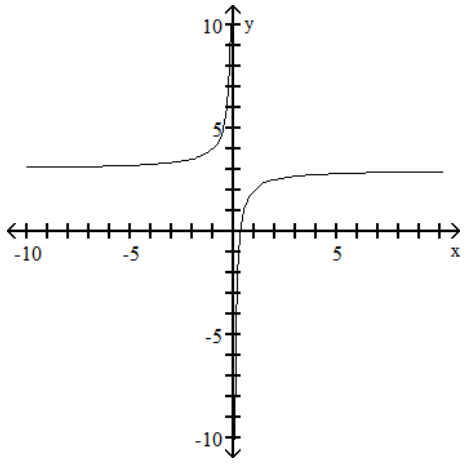


A) $y = |x + 5|$ B) $y = |x| - 5$

C) $y = |x - 5| + 1$ D) $y = |x - 5|$

Answer: D

149)



A) $y = -\frac{1}{x} + 3$

B) $y = -\frac{1}{x} - 3$

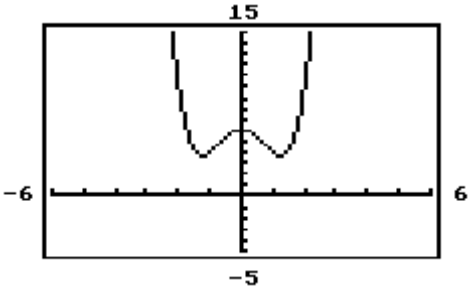
C) $y = -\frac{1}{x}$

D) $y = \frac{1}{x} + 3$

Answer: A

Match the given graph with its polynomial function.

150)



A) $f(x) = x^6 - 2x^2 - 6$

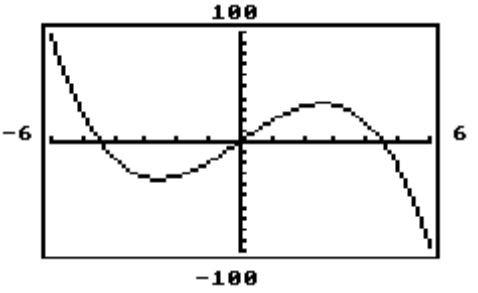
B) $f(x) = x^4 - 3x^2 + 6$

C) $f(x) = x^5 + 2x^3 + 6$

D) $f(x) = -x^5 - 3x^2 + 6$

Answer: B

151)



A) $f(x) = x^3 - 4x^2 + 20x$

B) $f(x) = x^3 + 9x$

C) $f(x) = -x^3 + 4x^2 - 9$

D) $f(x) = -x^3 + 20x$

Answer: D